Confounding in Factorial Designs

In factorial experiments, as the number of factors and the levels at which they are employed increase, the total number of treatment combinations increases rather rapidly and consequently the block size has to be enlarged.

For example, for a 2^{10} factorial experiment, a complete factorial would require 1024 units. A large scale experiment of this magnitude may involve a number of blocks and different treatments. The heterogeneity introduced as a consequence of the size of experiment results in extraneous variations which will add to experimental error.

Increase in the block size or handling such a huge experiment, the purpose of local control is defeated due to the following reasons.

(i) It is sometimes impracticable to get one complete replicate units which are relatively homogeneous and

(ii) The greater ~~homo~~ hetero geneity is introduced in the experimental error.

Hence the precision of a factorial experiment is affected if the treatment combinations are large in number. In order to maintain the homogeneity within the blocks, the experimenter must either cut down the number of factors or use an incomplete factorial which investigates the main effect of the factors by suitably sub-dividing the experimental material into smaller homogeneous blocks. The heterogeneity of blocks is allowed to affect only interactions which are likely to be unimportant.

The process by which unimportant comparision are deliberately confused or mixed up with the block comparisions, for the purpose of assessing more important comparisions with greater precision is called <u>Confounding</u>.

Confounding may also be defined as the technique of reducing the size of a replication over a number of blocks at the cost of losing some information on some effect which is not of much practical importance.

<u>Table of signs and divisors giving M and factorial effects in terms of treatment means for 2^3-design are as follows</u>:

Factorial Effect	Treatment Mean								Divisor
	(1)	(a)	(b)	(ab)	(c)	(ac)	(bc)	(abc)	
M	+	+	+	+	+	+	+	+	8
A	−	+	−	+	−	+	−	+	4
B	−	−	+	+	−	−	+	+	4
C	−	−	−	−	+	+	+	+	4
AB	+	−	−	+	+	−	−	+	4
AC	+	−	+	−	−	+	−	+	4
BC	+	+	−	−	−	−	+	+	4
ABC	−	+	+	−	+	+	−	+	4

Confounding in 2^3 Experiments :

In a 2^3-Expt, the eight treatment combinations require 8 units of homogeneous material each to form a block. If we decide to use blocks of 4 units (plots) each then a full replication will require only two blocks. In this case 8 treatment combinations are divided into two groups of 4 treatments each in a special way so as to confound any one of the less important interactions with blocks and these groups are allocated at random in the two blocks.

For example, let us consider confounding the highest order interactions ABC. We know that interaction effect to ABC is given by

$$ABC = \frac{1}{4}\left[(abc) - (bc) - (ac) + (c) - (ab) + (b) + (a) - (1)\right]$$

$$= \frac{1}{4}\left[(abc) + (a) + (b) + (c) - (ab) - (ac) - (bc) - (1)\right] \rightarrow (1)$$

Thus in order to confound the interaction ABC with blocks all the treatment combinations with positive signs are allocated at random in one block and those with negative signs in the other blocks. Thus, the arrangement will be

Replicate	Block 1 : (1) (ab) (ac) (bc)
	Block 2 : (a) (b) (c) (abc)

in the above Table gives ABC confounded with blocks and hence we lose information on ABC.

From (1), we observe that the constrast estimating ABC also contains block effect, effect of block 1 minus the effect of block 2.

The other six factorial effect which are also contrasts, viz., A, B, C, AB, AC, BC each contains two treatments in block 1 (or 2) with positive signs and two with negative signs so that they are orthogonal with block totals and

Hence these differences are not influenced by differences among blocks and can thus be estimated and tested as usual without any difficulty.

For carrying out statistical analysis, the various factorial effects and their S.S are estimated in the usual manner by using yate's method with the modification that neither the S.S due to the confounded interaction is computed nor it is included in the ANOVA table.

This confounded component is contained in the $(2r-1)$ d.f due to blocks. The d.f in the ANOVA table will be as given below.

S.V	d.f
Blocks	$(2r-1)$
A	1
B	1
C	1
AB	1
AC	1
BC	1
~~ABC~~	
Error	$6(r-1)$
Total	$8r-1$

ESS is obtained as usual by subtraction,

i.e., $ESS = T.S.S - S_A^2 - S_B^2 - S_C^2 - S_{AB}^2 - S_{AC}^2 - S_{BC}^2$.